INFOR, Vol. 46, No. 2, May, pp. 147–153
ISSN 0315-5986 | EISSN 1916-0615

An Economic Model for Justifying the Reduction of Delivery Variance in an Integrated Supply Chain

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Abstract—This paper addresses the economic impact of improving delivery performance in a serial make-to-order supply chain when delivery performance is evaluated with respect to a delivery window. Building on contemporary management theories that advocate variance reduction as the critical step in improving the overall performance of a system, an expected cost model is developed that financially quantifies the benefit of reducing delivery variance to the final customer in a serial supply chain. The objective of the model is to determine the variance level that minimizes the costs associated with untimely delivery (expected earliness and lateness) and the investment cost required for reducing the delivery variance. A logarithmic investment function is used and the model solution involves the minimization of a convex-concave total cost function. Numerical examples are provided to illustrate the model and the solution procedure.

Keywords Improving delivery performance, justifying variance reduction, supply chain management.

1. INTRODUCTION

In today's competitive business environment, customers require dependable on-time delivery from their suppliers. Recent empirical research has identified delivery performance as a key management concern among supply chain practitioners (see for example [1], [2], [3]). A conceptual framework for defining delivery performance in supply chain management is found in [4]. Within this framework, delivery performance is classified as a strategic level supply chain performance measure while delivery reliability is viewed as a tactical level supply chain performance measure. The framework suggested in [4] advocates that to be effective, supply chain management tools, delivery performance and delivery reliability need to be measured in financial (as well as non-financial) terms.

Failure to quantify delivery performance in financial terms presents both short-term and long-term difficulties. In the short term, the buyer-supplier relationship may be negatively impacted. A norm value of "presumed" performance is established by default when delivery performance is not formally measured [5]. This norm value stays constant with time and is generally higher than the organization's actual delivery performance.

It has been demonstrated that supplier evaluation systems have a positive impact on the buyer-supplier relationships, with these relationships ultimately having a positive impact on financial performance [6]. In the long term, failure to measure supplier delivery performance in financial terms may impede the capital budgeting process, which is necessary in order to support the improvement of supplier operations within a supply chain.

Delivery lead time is defined to be the elapsed time from the receipt of an order by the originating supplier in the supply chain to the receipt of the product ordered by the final customer in the supply chain. Delivery lead time is composed of a series

Received 21 November 2006, Revision 16 May 2008, Accepted 31 July 2008.

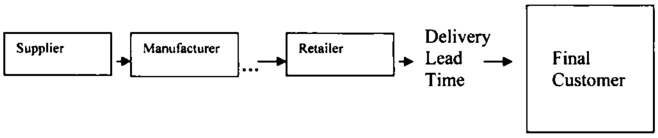

Figure 1. An n-stage serial supply chain

of internal (manufacturing and processing) activity times at each stage plus the external (distribution and transportation) lead times found at various stages of the supply chain. Figure 1 illustrates the delivery lead time components of a serial supply chain.

Early and late deliveries introduce waste in the form of excess cost into the supply chain; early deliveries contribute to excess inventory holding costs, while late deliveries may contribute to production stoppages costs, lost sales and loss of goodwill. To protect against untimely deliveries, supply chain managers often inflate in process inventory levels and production flow buffers. These actions can contribute to excess operating costs [7].

A review of 50 delivery evaluation models identified several shortcomings in modeling delivery performance [8]. These concerns are three-fold. First, delivery performance measures are not cost-based. Second, delivery performance measures ignore variability. Third, delivery performance measures often fail to take into account the penalties associated with both early and late deliveries. The inability to translate delivery performance into financial terms which incorporates uncertainty as well as realistically quantifying delivery timeliness (early as well as late delivery) hinders management's ability to justify capital investment for continuous improvement programs to improve delivery performance. The current research presented herein attempts to overcome these limitations through the development of a cost-based model that incorporates delivery variability and assigns penalties for both early and late deliveries.

In this paper, we develop a cost-based performance metric for evaluating delivery performance and reliability to the final customer in a serial make-to-order supply chain that is operating under a centralized management structure. Contemporary management theories advocate the reduction of variance as a key step in improving the performance of a system [9], [10], [11]. In union with these prevailing theories, delivery performance is modeled as a cost-based function of the delivery variance. The financial benefit of reducing variability in delivery performance is demonstrated by the model.

This paper is organized as follows. In Section 2, an analytical model based on the expected costs associated with untimely delivery is developed and propositions are introduced to analyze the model in terms of the variance. In Section 3, improvement in delivery performance is modeled using a logarithmic investment function. The reduction of delivery variance is formulated as a mathematical optimization problem that is characterized by a convex-concave total cost function. A solution procedure is introduced and numerical experiments are conducted. In the concluding section, we summarize the findings of this research and present directions for future research.

2. MODEL DEVELOPMENT

Consider an n-stage serial make-to-order supply chain operating under a centralized management structure where an activity at each stage contributes to the overall delivery time to the final customer. The activity at a given stage is defined to be the duration of time exhausted at that stage of the supply chain. The duration of time experienced at a given stage may consist of the manufacturing time to meet a lot size requirement, the time to draw materials from an inventory to lot size requirements, or a combination of production and draw from inventory.

The activity duration of stage i, W_i, is represented by probability density function $f_W(w, \theta)$ that is reproductive under addition with respect to parameter set θ. Delivery time to the final customer, $X = \sum_{i=1}^{n} W_i$, is defined by the probability density function $f_X\left(x, \sum_{i=1}^{n} \theta_i\right)$. We assume that activity durations at each stage of the supply chain are normally distributed and independent. Normality and independence among stages is often assumed in the literature [12], [13]. When activity times at stage i are normally distributed and independent, delivery time to the final customer is defined as a result of the n-fold convolution of $f_W(w, \theta)$ which yields $X \sim N\left(x; \sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} v_i\right)$ where μ_i and v_i are the delivery mean and variance for stage i, $i \in [1, n]$.

Delivery to the final customer is analyzed with regard to the customer's specification of delivery timeliness as defined by a delivery window. Delivery windows are an effective tool for modeling the expected costs associated with untimely delivery. Several researchers advocate the use of delivery windows in time-based manufacturing systems [14], [15], [16]. Metrics based on delivery (order) windows capture the most important aspect of the delivery process, which is reliability [17]. Under the concept of a delivery window, the customer supplies an earliest allowable delivery date and a latest allowable delivery date. A delivery window is defined as the difference between the earliest acceptable delivery date and the latest acceptable delivery date. Within the delivery window, a delivery may be classified as early, on-time, or late (see Figure 2). Delivery lead time, X, is a random variable with probability density function $f_X(x)$. The on-time portion of the delivery window is defined by $c_2 - c_1$. Ideally, $c_2 - c_1 = 0$. However, the extent to which $c_2 - c_1 > 0$ may be measured in hours, days, or weeks depending on the industrial situation.

This paper assumes the following: (1) delivery performance is stable enough so that the modal delivery time is within the

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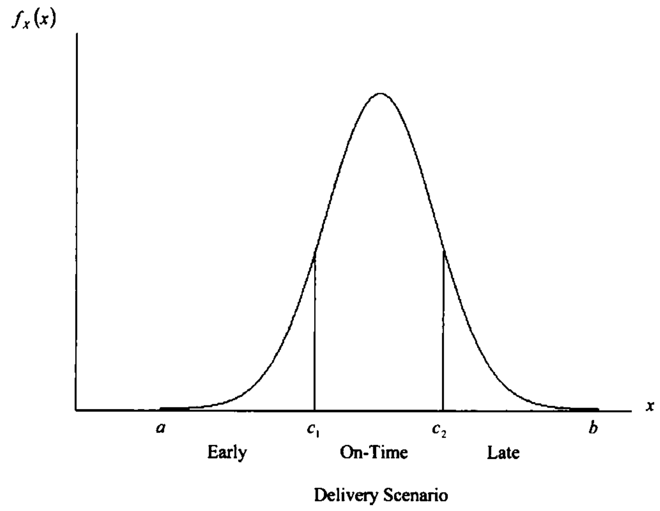
ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.2.147



Legend:
a = earliest delivery time
c_1 = beginning of on-time delivery
c_2 = end of on-time delivery
b = latest acceptable delivery time

Figure 2. Illustration of delivery window

on-time portion of the delivery window, (2) the mean and on-time portion of the delivery window remain fixed, and (3) the coefficient of variation of X is less than 0.25. For situations necessitating the need to truncate the normal density to prevent nonnegative delivery times or select symmetrical and nonsymmetrical density functions defined for only positive values of the delivery time see [8], [18].

Consider an n-stage serial supply chain in operation over a time horizon of length T years, where a demand requirement of the final customer for a single product of D units will be met with a constant delivery lot size Q. The expected penalty cost per delivery period for untimely delivery, Y, is

$$Y = QH\int_a^{c_1}(c_1 - x)f_X(x)dx + K\int_{c_2}^{b}(x - c_2)f_X(x)dx \quad (1)$$

where $Q =$ constant delivery lot size per cycle, $H =$ inventory holding cost per unit per time, $K =$ penalty cost per time unit late (levied by the final customer), $a, b, c_1, c_2 =$ parameters defining the delivery window $W_j =$ time duration of activity $j(j = 1, 2, \ldots, n)$, $f_X(x) = f_{W_1+W_2+\cdots+W_n} =$ density function of delivery time X.

It is a common purchasing agreement practice to allow the buyer (final customer) to charge suppliers for untimely deliveries (see for example [19], [20]). Reductions in early deliveries reduced inventory holding costs at Hewlett-Packard by \$9 million [21]. It has been reported in the automotive industry Saturn levies fines of \$500 per minute against suppliers who cause production line stoppages [22] and that Chrysler fines suppliers \$32,000 per hour when an order is late [23]. The penalty cost in these cases is an opportunity cost due to lost production. Purchasing managers often view the production disruptions caused by delivery stockouts to be more widespread and more costly than the lost sales that stockouts cause [24]. Hence, K has been defined as an opportunity cost due to lost production [22], [23].

Evaluating (1) under the defined assumptions yields the total expected penalty cost (see Appendix A for derivation)

$$Y = QH\left[\sqrt{v}\phi\left(\frac{c_1-\mu}{\sqrt{v}}\right) + (c_1-\mu)\left(\Phi\left(\frac{c_1-\mu}{\sqrt{v}}\right)\right)\right] + K\left[\sqrt{v}\phi\left(\frac{c_2-\mu}{\sqrt{v}}\right) - (c_2-\mu)\left(1-\Phi\left(\frac{c_2-\mu}{\sqrt{v}}\right)\right)\right]. \quad (2)$$

Proposition 1. *The expected penalty cost is a monotonically increasing non-convex function of the variance.*

Proof. The first and second derivatives of Y with respect to the variance are respectively

$$Y'(v) = \frac{QH\phi((c_1-\mu)/\sqrt{v}) + K\phi((c_2-\mu)/\sqrt{v})}{2\sqrt{v}} \quad (3)$$

and

$$Y''(v) = \frac{QH}{4v^{5/2}}\left\{\phi\left(\frac{c_1-\mu}{\sqrt{v}}\right)((c_1-\mu)^2 - v)\right\} + \frac{K}{4v^{5/2}}\left\{\phi\left(\frac{c_2-\mu}{\sqrt{v}}\right)((c_2-\mu)^2 - v)\right\}. \quad (4)$$

■

Examining (3), the expected penalty cost is an increasing function of the variance since $Y'(v) > 0$ for positive values of Q, H, K and v. Examining (4), we note that $Y''(v) > 0$ when $v \le \min\{(c_1-\mu)^2, (c_2-\mu)^2\}$, but $Y''(v) < 0$ when $v \ge \max\{(c_1-\mu)^2, (c_2-\mu)^2\}$, hence the expected penalty cost is not a convex function of the variance.

3. MODELING IMPROVEMENT IN DELIVERY PERFORMANCE

Ideally the expected costs incurred for untimely delivery should be equal to zero. This implies that, for the currently defined delivery window, all deliveries are within the on-time portion of the delivery window, and that waste in the form of early and late deliveries has been eliminated from the system.

For a fixed mean and delivery window, the expected penalty cost can be reduced by reducing the variance of the delivery distribution. Reducing the variance of delivery can be achieved by initiating improvements such as: (i) gaining tighter control over process flow times at downstream stages of the supply

INFOR, Vol. 46, No. 2, May 2008, pp. 147–153
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chain, (ii) enhanced coordination of freight transport, (iii) more efficient material handling of inbound and outbound stock, and (iv) improved communications among stages in the supply chain such as implementing electronic data interchange (EDI). Initiating such improvements requires capital investment (see for example, [25]).

An optimization model which considers the delivery variance as a decision variable is defined. The objective of the model is to determine the variance level that minimizes the costs (expected earliness and lateness) associated with untimely delivery and the investment cost required for reducing the delivery variance.

The optimization problem is

$$\text{Minimize } G(v) = Y(v) + C(v) \tag{5}$$

where v = variance of delivery distribution, $Y(v)$ = expected penalty cost due to untimely delivery, $C(v)$ = investment required for a delivery variance of v.

A logarithmic investment cost function is used to model the cost of reducing the delivery variance. Under this investment function, reducing the delivery variance by a fixed percentage requires a fixed amount of investment. This functional form is appealing in that each additional reduction in the delivery variance is more costly than the previous reduction. The logarithmic investment function has been widely adopted in the literature (see for example [26], [27], [28]).

Let v_0 equal the current value of delivery variance and λ represent the cost of reducing the delivery variance by h percent. The investment function is then

$$C(v) = \frac{\lambda}{\ln(1/1-h)}[\ln(v_0) - \ln(v)] \quad \text{for } 0 \le v \le v_0. \tag{6}$$

Proposition 2. The investment function is a decreasing convex function of the delivery variance.

Proof. The first and second derivatives of (6) are

$$C'(v) = -\frac{\lambda}{\ln(1/1-h)v} \tag{7}$$

and

$$C''(v) = \frac{\lambda}{\ln(1/1-h)v^2}. \tag{8}$$

We observe that $C'(v) < 0$ and $C''(v) > 0$ for $\lambda > 0$ and $h > 0$. ■

The optimization model is

Minimize

$$\begin{aligned} G(v) = {} & QH\left[\sqrt{v}\phi\left(\frac{c_1-\mu}{\sqrt{v}}\right) + (c_1-\mu)\left(\Phi\left(\frac{c_1-\mu}{\sqrt{v}}\right)\right)\right] \\ & + K\left[\sqrt{v}\phi\left(\frac{c_2-\mu}{\sqrt{v}}\right) - (c_2-\mu)\left(1-\Phi\left(\frac{c_2-\mu}{\sqrt{v}}\right)\right)\right] \\ & + \frac{\lambda}{\ln(1/1-h)}[\ln(v_0) - \ln(v)]. \end{aligned} \tag{9}$$

Per Proposition 1, $Y(v)$ is a convex function of the variance for $v \le \min\{(c_1-\mu)^2, (c_2-\mu)^2\}$ and a concave function when $v \ge \max\{(c_1-\mu)^2, (c_2-\mu)^2\}$.

For $\min\{(c_1-\mu)^2, (c_2-\mu)^2\} < v < \max\{(c_1-\mu)^2, (c_2-\mu)^2\}$ the convexity-concavity of $Y(v)$ is parameter specific. Hence, $G(v)$ is the sum of a convex-concave function $Y(v)$ and a convex function $C(v)$.

Theorem 1. *$G(v)$ is a convex function of the delivery variance provided $v \le \min\{(c_1-\mu)^2, (c_2-\mu)^2\}$.*

Proof. For $v \le \min\{(c_1-\mu)^2, (c_2-\mu)^2\}$, $G(v)$ is the sum of two convex functions (see Propositions 1 and 2) and is hence convex. ■

Corollary 1. $\tilde{G}(v)$ is minimized at $G^*(v^*)$ provided $v^* \le v_m$.

Proof. Let $v_m = v \le \min\{(c_1-\mu)^2, (c_2-\mu)^2\}$. Per Theorem 1, $G(v)$ is a convex function of the delivery variance for $v \le v_m$, hence the solution of $G'(v) = 0$ yields an optimal solution if $v^* \le v_m$. ■

The value of v^* that minimizes $G(v)$ at $G^*(v^*)$ does not exist in closed form and must be found by numerically solving $G'(v) = 0$ for $v = v^*$. Solving (10) for v takes the general form of

$$\sqrt{v}\left[QH\phi\left(\frac{c_1-\mu}{\sqrt{v}}\right) + K\phi\left(\frac{c_2-\mu}{\sqrt{v}}\right)\right] - \frac{2\lambda}{\ln(1/(1-h))} = 0. \tag{10}$$

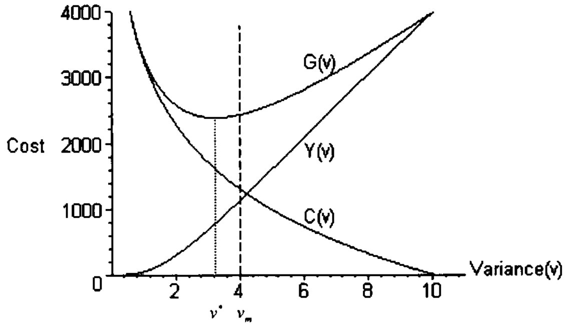

Figure 3. Optimal solution to numerical illustration with $v < v_m$

INFOR, Vol. 46, No. 2, May 2008, pp. 147–153
ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.2.147

TABLE 1
Summary of numerical examples with $v > v_m$

Width of Delivery Window	Variance Reduction Cost λ	Variance Reduction Rate h	v_m	LINGO Solution v	LINGO Solution $G(v)$	Number of Iterations
2 Days	\$500	0.10	1.00	6.588	\$11,277	135
	500	0.15	1.00	3.239	8,861	139
	650	0.10	1.00	10.520	12,513	133
	650	0.15	1.00	4.925	10,342	136
3 Days	500	0.10	2.25	7.608	9,679	141
	500	0.15	2.25	4.111	7,574	119
	650	0.10	2.25	11.611	10,745	118
	650	0.15	2.25	5.890	8,864	126
4 Days	500	0.10	4.00	8.881	8,323	131
	500	0.15	4.00	5.163	6,532	132
	650	0.10	4.00	13.009	9,120	132
	650	0.15	4.00	7.076	7,634	125

Introducing $\phi(z) = (1/\sqrt{2\pi})\exp(-z^2/2)$ gives,

$$\sqrt{v}\left[QH\exp\left(-\frac{(c_1-\mu)^2}{2v}\right) + K\exp\left(-\frac{(c_2-\mu)^2}{2v}\right)\right] - \frac{2\lambda\sqrt{2\pi}}{\ln(1/(1-h)(1-h))} = 0. \quad (11)$$

If the on-time portion of the delivery window is symmetric then $\delta = c_2 - \mu = -(c_1 - \mu)$ and (11) simplifies to

$$\sqrt{v}\left[\exp\left(-\frac{\delta^2}{2v}\right)\right] - \frac{2\lambda\sqrt{2\pi}}{\ln(1/(1-h))(QH+K)} = 0. \quad (12)$$

A closed form solution for (12) is

$$v = \frac{\delta^2}{W_L(\delta\ln(1/(1-h))(QH+K)/2\lambda\sqrt{2\pi})^2} \quad (13)$$

where W_L is the Lambert W function. The Lambert W function is used to obtain closed form solutions for functional equations that involve exponentials. The Lambert W is described in detail in [29] and is found in standard optimization software packages such as Maple [30].

Numerical Examples

Consider a supply chain where delivery time (in days) to the final customer is normally distributed with a mean of 50 and variance of 10. The on-time portion of the delivery window is defined by $c_1 = 48$ and $c_2 = 53$. For these parameters $v_m = 4$ and $v_0 = 10$. Additional parameters for the expected penalty cost model are: $Q = 500$, $H = \$10$ and $K = \$5{,}000$. Under the logarithmic form of the investment function, a cost of $\lambda = \$150$ is incurred for every $h = 10\%$ reduction in the delivery variance. Solving (9) for v yields $v = 3.253$. Per Corollary 1 of Theorem 1, $v = 3.253 < v_m = 4$, hence $v = v^* = 3.253$ and $G^*(v^*) = \$2387$ (see Figure 3).

When $v \geq \max\{(c_1-\mu)^2, (c_2-\mu)^2\}$, minimizing $G(v)$ requires the minimization of the sum of a concave function $Y(v)$ and a convex function $C(v)$. There is no guarantee that solving $G'(v) = 0$ will yield the optimal value of the variance. Detailed numerical analyzes (see [18]) have indicated that non-convexity in the expected penalty cost term $Y(v)$ when $v \geq \max\{(c_1-\mu)^2, (c_2-\mu)^2\}$ is very slight and that very good solutions to the minimization of $G(v)$ can be obtained using commercially available optimization software such as LINGO [31].

The results of twelve sample problems solved using LINGO are presented in Table 1. The on-time portion of the delivery window was varied from 2 days (one day early, one day late) to 4 days (two days early, three days late) when the delivery distribution to the final customer was normally distributed with a mean of 50 and a variance of 20. For these parameters $v_m = 1.00$, 2.25, and 4.00 and $v_0 = 20$. Under the logarithmic form of the investment function, variance improvement costs of $\lambda = \$500$ and \$650 were studied with variance reduction rates

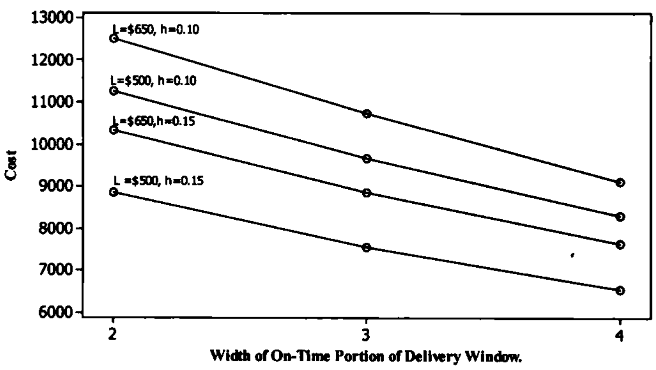

Figure 4. Sensitivity of solutions to model parameters

INFOR, Vol. 46, No. 2, May 2008, pp. 147–153
ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.2.147

of $h = 10\%$ and 15%. Additional parameters used were: $Q = 500$, $H = \$10$ and $K = \$5{,}000$. All solutions were obtained in less than 0.01 seconds. The sensitivity of the solutions to the width of the on-time portion of the delivery window, variance improvement cost and variance reduction rate is illustrated in Figure 4.

4. CONCLUSIONS

This paper addressed one aspect of supply chain planning by modeling delivery performance to the final customer in a serial make-to-order supply chain from the contemporary perspective of reducing delivery variability. A cost-based model has been presented that financially evaluates the effects of reducing delivery variability on overall delivery performance. The model was demonstrated under a logarithmic investment function and a framework for determining optimal and near optimal levels of delivery variance was developed. As demonstrated through a numerical example, the results of this modeling may prove useful to justifying the investment required to reduce delivery variance to a targeted goal as a part of an overall continuous improvement program to improve supply chain operating performance.

There are several aspects of this research that could be expanded. First, the assumption of independence of the activity times among the stages could be investigated. Second, an industrial case study utilizing the model developed herein could be conducted. Third, the model could be generalizing by introducing probability density functions other than the normal to model activity times at various stages of the supply chain. Fourth, this paper addressed the case of make-to-order, and therefore the model presented herein may be extended to the case of make-to-stock by adding an inventory component. Lastly, the scope of the model could be expanded by allowing disruptions in the variance reduction process.

ACKNOWLEDGEMENTS

The authors are grateful to the two anonymous referees and to the Editor of the Journal for their helpful comments and suggestions. M.Y. Jaber thanks the Natural Sciences and Engineering Research Council of Canada (NSERC) for supporting his research.

APPENDIX A. DERIVATION OF EXPECTED PENALTY COST (EQUATION 2)

The density function of delivery time X with mean μ and variance v, is the convolution of the normally distributed lead time components $W_i (i = 1, 2, \ldots n), f_X(x) = f_{W_1+W_2+\cdots+W_n}$. If an earliest delivery time (a) and latest acceptable delivery time (b) are imposed on $f_X(x)$, then

$$h_X(x) = \frac{f_X(x)}{\int_a^b f_X(x)} \tag{A.1}$$

and (1) is

$$Y = QH\int_a^{c_1}(c_1 - x)h_X(x)dx + K\int_{c_2}^{b}(x - c_2)h_X(x)dx. \tag{A.2}$$

Examining (A.2), we observe that Y is separable in terms of the expected earliness cost and the expected lateness cost.

Expected Lateness Cost

The expected penalty cost for late delivery is

$$Y_{late} = \frac{K}{p}\int_{c_2}^{b}(x - c_2)f_X(x)dx \tag{A.3}$$

where

$$p = \int_a^b f_X(x)dx \tag{A.4}$$

$$Y_{late} = \frac{K}{p}\left[\int_{c_2}^{b}\frac{x}{\sqrt{2\pi v}}\exp\left\{\frac{-(x-\mu)^2}{2v}\right\}dx - \int_{c_2}^{b}\frac{c_2}{\sqrt{2\pi v}}\exp\left\{\frac{-(x-\mu)^2}{2v}\right\}dx\right]. \tag{A.5}$$

Substituting $z = (x - \mu)/\sqrt{v}, x = \sqrt{v}z + \mu$, and $dx = \sqrt{v}dz$ into (A.5) and simplifying yields,

$$Y_{late} = \frac{K}{p}\left[\sqrt{v}\int_{(c_2-\mu)/\sqrt{v}}^{(b-\mu)/\sqrt{v}}\frac{z}{\sqrt{2\pi}}\exp\{-z^2/2\}dz + (\mu - c_2)\int_{(c_2-\mu)/\sqrt{v}}^{(b-\mu)/\sqrt{v}}\frac{1}{\sqrt{2\pi}}\exp\{-z^2/2\}dz\right]. \tag{A.6}$$

Introducing $\phi(\cdot)$ and $\Phi(\cdot)$ as the standard normal density (ordinate) and cumulative distribution functions respectively, and recognizing that for the standard normal that $\int_w^\infty zf(z) = \phi(w)$, gives

$$Y_{late} = \left[\frac{K}{\Phi\left(\frac{b-\mu}{\sqrt{v}}\right) - \Phi\left(\frac{a-\mu}{\sqrt{v}}\right)}\right]\left[\sqrt{v}\left(\phi\left(\frac{c_2-\mu}{\sqrt{v}}\right) - \phi\left(\frac{b-\mu}{\sqrt{v}}\right)\right) - (c_2 - \mu)\left(\Phi\left(\frac{b-\mu}{\sqrt{v}}\right) - \Phi\left(\frac{c_2-\mu}{\sqrt{v}}\right)\right)\right]. \tag{A.7}$$

Expected Earliness Cost

Repeating the steps outlined in (A.3 – A.7) for the expected earliness cost

$$Y_{early} = \frac{QH}{p}\int_a^{c_1}(c_1 - x)f_X(x)dx \tag{A.8}$$

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.2.147

yields

$$Y_{early} = \left[\frac{QH}{\Phi\left(\frac{b-\mu}{\sqrt{v}}\right) - \Phi\left(\frac{a-\mu}{\sqrt{v}}\right)}\right]\left[\sqrt{v}\left(\phi\left(\frac{c_1-\mu}{\sqrt{v}}\right) - \phi\left(\frac{a-\mu}{\sqrt{v}}\right)\right)\right.$$
$$\left.-(c_1-\mu)\left(\Phi\left(\frac{a-\mu}{\sqrt{v}}\right) - \Phi\left(\frac{c_1-\mu}{\sqrt{v}}\right)\right)\right]. \qquad \text{(A.9)}$$

Negative values for the normal distribution are negligible provided $\mu > 4\sqrt{v}$, hence we set $b = \mu + 4\sqrt{v}$ and $b = \mu + 4\sqrt{v}$. This implies $\phi((a-\mu)/\sqrt{v}) = \phi((b-\mu)/\sqrt{v}) \cong 0, \Phi((a-\mu)/\sqrt{v}) \cong 0.0$ and $\Phi((b-\mu)/\sqrt{v}) \cong 1.0$.

Combining (A.7) and (A.9) and simplifying gives

$$Y = QH\left[\sqrt{v}\phi\left(\frac{c_1-\mu}{\sqrt{v}}\right) + (c_1-\mu)\left(\Phi\left(\frac{c_1-\mu}{\sqrt{v}}\right)\right)\right]$$
$$+K\left[\sqrt{v}\phi\left(\frac{c_2-\mu}{\sqrt{v}}\right) - (c_2-\mu)\left(1-\Phi\left(\frac{c_2-\mu}{\sqrt{v}}\right)\right)\right]. \qquad \text{(A.10)}$$

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INFOR, Vol. 46, No. 2, May 2008, pp. 147–153
ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.2.147